Domain Decomposition Method and Model Order Reduction Method for Electromagnetic-Thermal Coupled Problem

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A numerical simulation for electromagnetic heat induction is performed. Domain decomposition method is combined with model order reduction for the calculation of electromagnetic thermal coupled problem. Model order reduction (MOR) based on Proper Orthogonal Decomposition (POD) is used to find a low-order approximation to the original high order problems such that the computation could be improved effectively. Also a non-overlapping domain decomposition method is used to partition the calculation domain into two subdomains for simplicity. A serial staggered coupling strategy is used to solve the coupled problem. The simulation result is verified with that of simulation case contained in the COMSOL application library. The aim of this paper is to provide an approach to promote the computation efficiency of Multiphysics simulation.

***Index Terms*—Coupled problem, domain decomposition method, model order reduction, proper orthogonal decomposition.**

I. Introduction

THE COUPLED problems are quite prevalent in the natural environment and industrial production. As the industrial products are more and more compact, elaborate, the multiphysics phenomena are crucial for high performance industrial products' design.

As it is known, there are two main strategies for the calculation of coupled problems: monolithic method and partitioned method. The monolithic way treats the whole coupled problem as an entity, and all sub problems are advanced and solved each time-step synchronously [1]. The partitioned approach deals with the coupled problems by adoption the divide and conquer strategy. The coupled problems are divided into separated sub problems physically or subdomains dimensionally [2]. Obviously, the monolithic way are more close to real world coupled phenomena. However, it is only applicable for specific problem and there is no standard codes. What is more, it may forms a large indefinite matrix for 3D problems. And the comparable length/time scale circumstance favors a monolithic way. Hence, for the electromagnetic heat induction (EMHI) problem, it is advisable to take a partitioned strategy to solve the problem as the time scale of the single physics are distinctive.

In order to promote the computation efficiency of coupled problems, a domain decomposition method (DDM) is combined with model order reduction (MOR) to solve the EMHI problem in a serial staggered way. Kerfriden conducted a research on nonlinear fracture mechanics by coupling the DDM and projected based MOR to rationalize the computational expense [3]. Corigliano studied the dynamic elastic-plastic structural problem using DDM and MOR with computation time reduced about 50%, while maintaining the accuracy [4]. However, the studies above is not coupled problem application. The applications of DDM and MOR for coupled problems are very limited. In [5], a simulation on coupled electro-mechanical problems in micro electro-mechanical systems is performed and results with a significant computational gains up to 98%. However, it only involves the surface coupling and comparable time scale. Hence, this paper is aimed to extend the DDM and MOR to volume coupling and different timescale coupled problems.

The paper is organized as follows: section ii is mainly described the governing equations, DDM and MOR for the coupled problem. The implementation for the DDM and MOR to the coupled problem. Serial staggered procedure for solving the coupled [problem is given in section iii. Results were shown in section 4. Finally, a conclusion is given.

II. Mathematical Models

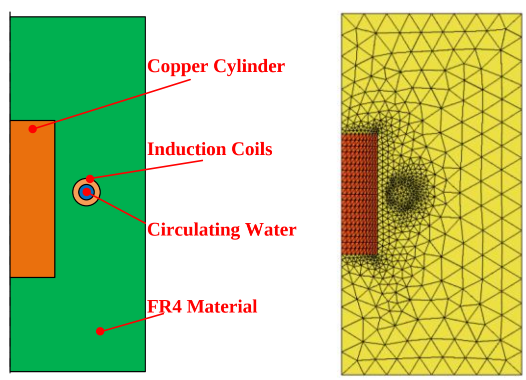

Fig.1. (a) Schematic of EMHI, and (b) Mesh for domain decomposition

A. *Physics description*

The schematic diagram of EMHI is shown in Fig.1.(a). The induced currents in a copper cylinder produce heat, and when the temperature rises, the electric conductivity of the copper changes. Solving the heat transfer simultaneously with the field propagation is therefore crucial for an accurate description of this process. A challenge in induction heating is that the high current in the induction coils requires active cooling. The circulating water inside the hollow induction coils is used for cooling. For mechanical support and electrical insulation, the cylinder and coil are embedded in FR4 composite material.

B. Governing equations

Governing equations for axisymmetric model in eddy current.

$$\frac{1}{\mu}\left[\frac{\partial}{\partial \boldsymbol{z}}\left(\frac{1}{\boldsymbol{r}}\frac{\partial(\boldsymbol{r}\dot{\boldsymbol{A}}_z)}{\partial \boldsymbol{z}}\right)+\frac{\partial}{\partial \boldsymbol{r}}\left(\frac{1}{\boldsymbol{r}}\frac{\partial(\boldsymbol{r}\dot{\boldsymbol{A}}_z)}{\partial \boldsymbol{z}}\right)\right]=\boldsymbol{j}\omega\gamma\dot{\boldsymbol{A}}_z \tag{1}$$

$$\rho C_p\frac{\partial T}{\partial t}=\frac{\partial}{\partial \boldsymbol{z}}\left(\frac{k}{r}\frac{\partial(rT)}{\partial \boldsymbol{z}}\right)+\frac{\partial}{\partial r}\left(\frac{k}{r}\frac{\partial(rT)}{\partial r}\right)+q \tag{2}$$

where, μ is the magnetic permeability, $\dot{\boldsymbol{A}}_z$ is magnetic potential, ρ is density, C_p is thermal capacity, k is thermal conductivity, and q is heat source. T is temperature.

C. Domain decompositions method and model order reduction

For simplicity, the calculation domain Ω is divided into two subdomains Ω_i (i=1,2), represented by different colors respectively.

The implementation for the Lagrange multiplier could be realized by the following iterative algorithm:

$$\begin{cases} Lx_i^{(k+1)}=f, \text{in } \Omega_i \\ x_i^{(k+1)}\big|_{\Gamma_j}=C_j, \text{ on } B_{[i,j]} \\ \boldsymbol{n}_i\bullet(\nabla x_i^{(k+1)})=\pm\lambda^{(k)}, \text{ on } B \end{cases}$$

where $B_{[i,j]}$ is the external boundary, and C_j is the Dirichlet boundary; B is the interface of subdomains Ω_i. It is assumed that the flux is conserved across the interface, B, hence $\lambda^{(k)}$ is opposite. And $\lambda^{(k)}$ will be updated every iteration until the convergence is satisfied.

To further speed up the calculations, besides the above mentioned DDM, a POD based MOR method is also introduced. The main idea of the POD is to reduce the large number of interdependent variables to a much smaller number of uncorrelated variables while maintaining as much as possible of the original problems. The orthogonal transformation to the basis of eigenvectors of the sample covariance matrix is conducted, and the data then are projected onto the subspace spanned by eigenvectors corresponding to the largest eigenvectors. The strong point of the POD is that it can be applied to the non-linear partial equations, which is common in many partial equations, especially for the coupled problems.

III. Serial Stagger Procedure for EMHI Coupling

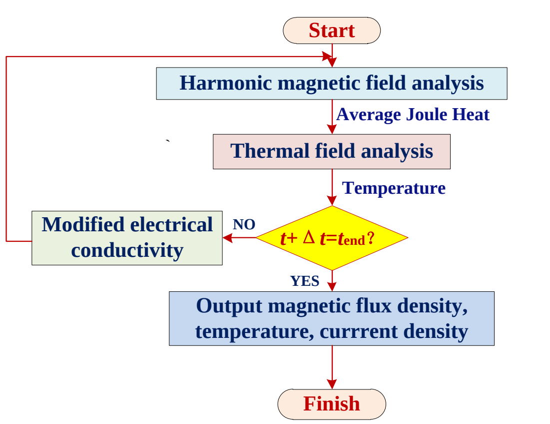

Fig. 2. Staggered coupling procedure.

The staggered coupling procedure of EMHI is depicted in the following picture. Firstly, harmonic magnetic field analysis is conducted at the very beginning of each time step and the average joule heat is obtained, which is used as the heat source for thermal field analysis in the rest of each time step. Then, the temperature distribution is acquired. The electrical conductivity is modified to restart harmonic magnetic field analysis, thermal field analysis repeatedly, until the pre-setting simulating time t_end is reached. Finally, output the values.

IV. Results and Discussions

A preliminary results are given in this part. The results are classical way to compute a coupled electromagnetic-thermal problem.

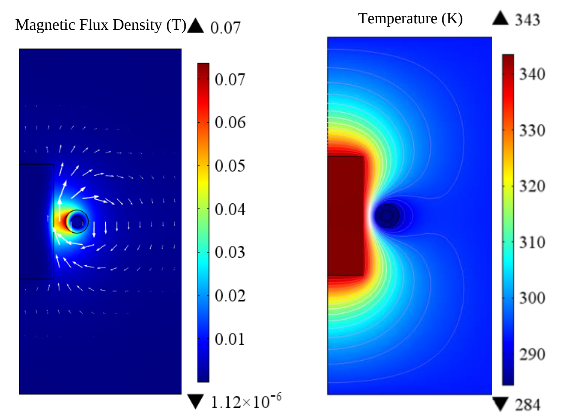

Fig. 3. (a) Magnetic flux density, and (b) temperature at t=36000s of the COMSOL simulation case

The simulation results calculated by the methods proposed in this paper are going to compare with the results shown in Fig. 3. And the computation efficiency will also be studied by taking the same mesh.

V. Conclusions

Domain decomposition method and model order reduction method are adopted to solve the volume coupling problems. It is hope that the computation efficiency could be promoted by the combination of domain decomposition method and model order reduction method.

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